QUASIDIFFUSION MODEL OF TRANSFER PROCESSES IN A GAP

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The problem of free-molecular flow in the gap between two plane parallel walls is reduced to solving the Poisson equation with an anisotropic tensor of transmission coefficients.

The problem of free-molecular transfer in a gap between two parallel surfaces arises in investigating the operation of vacuum cryogenic — and, in particular, multilayer — insulation. In view of the well-known analogy [1], the results obtained may also be used in considering radiant transfer.

In studying molecular transfer in long channels, the quasidiffusional approximation proposed in [2] is widely used. This method may be generalized to the problem of transfer in a narrow gap between parallel surfaces [3]. In the present work, various representations of the transmission tensor — the two-dimensional analog of the "diffusion" coefficient for long channels — are introduced and compared.

1. Formulation of the Problem

Consider a steady free-molecular flow in the gap between two plane parallel walls, each of which occupies a region S bounded by contour L on the plane.

It is assumed that diffuse emission is possible at the walls, while the reflection of the molecules from the walls is diffusional in character. At the side surface of the gap, i.e., at a cylindrical surface with directrix L and generatrices perpendicular to the walls, a diffuse flux density Q is incident from outside; Q depends on the position of the point on the contour and is homogeneous over the height of the gap.

The problem of determining the effective fluxes at the walls $q_j(\mathbf{y})$ ($j = 1, 2, \mathbf{y} \in S$) and the two-dimensional vector **G** of the mass flux density in the gap under the given assumptions reduces [3] to solving the integral equations

$$u(\mathbf{y}) = \int_S K(\mathbf{y}, \mathbf{y}')u(\mathbf{y}')dS_{y'} + \int_L K_0(\mathbf{y}, \mathbf{y}')Q(\mathbf{y}')dL_{y'} + u^*, \quad (1)$$

$$v(\mathbf{y}) = -\int_S K(\mathbf{y}, \mathbf{y}')v(\mathbf{y}')dS_{y'} + v^*, \quad (2)$$

where

$$u = q_1 + q_2;\ v = q_2 - q_1;\ u^* = \overset{*}{q}_1 + \overset{*}{q}_2;\ v^* = \overset{*}{q}_2 - \overset{*}{q}_1;$$

$$K(\mathbf{y}, \mathbf{y}') = \frac{h^2}{\pi[h^2 + (\mathbf{y} - \mathbf{y}')^2]^2};\ K_0(\mathbf{y}, \mathbf{y}') = \frac{h^2(\mathbf{n}, \mathbf{y}' - \mathbf{y})}{\pi[h^2 + (\mathbf{y} - \mathbf{y}')^2](\mathbf{y} - \mathbf{y}')^2}. \quad (3)$$

The vector **G**, averaged over the height of the gap, is expressed in terms of the function $u(\mathbf{y})$ by means of the formula

$$\mathbf{G} = \frac{h}{2\pi}\int_S \frac{u(\mathbf{y}')(\mathbf{y} - \mathbf{y}')}{[h^2 + (\mathbf{y} - \mathbf{y}')^2](\mathbf{y} - \mathbf{y}')^2} dS_{y'} + \frac{1}{\pi}\int_L \frac{Q(\mathbf{y}')\left(\frac{\pi}{2} - \arctan\frac{|\mathbf{y} - \mathbf{y}'|}{h}\right)(\mathbf{n}, \mathbf{y}' - \mathbf{y})(\mathbf{y} - \mathbf{y}')}{|\mathbf{y} - \mathbf{y}'|^3} dL_{y'}. \quad (4)$$

2. Various Representations of the Transmission-Coefficient Tensor

Numerical solution of Eqs. (1) and (2) is simple. Only in the case of a small gap ($h \ll d$, d is the characteristic dimension of region S) does the kernel $K(\mathbf{y}, \mathbf{y}')$ become almost

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δ-like, which makes numerical solution with acceptable accuracy difficult. However, the almost local character of the integral operator when $h \ll d$ also offers the possibility of replacing the problem by a considerably simpler version, permitting analytical solution in a number of cases.

Thus, let $h \ll d$. In this case, the approximate solution $v^0 = v^*/2$ may be written at once for Eq. (2), giving a discrepancy of the order of h^2/d^2 after substituting into Eq. (2). It is simple to show, by means of Taylor-series expansion of the unknown function, that at a sufficiently large distance from the contour L in region S, Eq. (1) may be replaced by the following differential equation with an accuracy up to terms of order $O(h^2/d^2)$

$$h^2 \operatorname{div}(\Pi^{(1)} \operatorname{grad} u) = -u^*, \tag{5}$$

where

$$\Pi^{(1)} = \begin{Vmatrix} \frac{1}{2\pi}\int_0^{2\pi} \cos^2\psi f_1(\psi)\,d\psi & \frac{1}{2\pi}\int_0^{2\pi} \sin\psi\cos\psi f_1(\psi)\,d\psi \\ \frac{1}{2\pi}\int_0^{2\pi} \sin\psi\cos\psi f_1(\psi)\,d\psi & \frac{1}{2\pi}\int_0^{2\pi} \sin^2\psi f_1(\psi)\,d\psi \end{Vmatrix}; \tag{6}$$

$$f_1(\psi) = \frac{1}{2}\left[\ln\left(1+\frac{a^2}{h^2}\right) - \frac{a^2}{h^2+a^2}\right].$$

It may now be noted that differential Eq. (5) may be obtained from the mass-conservation law, which takes the form

$$h \operatorname{div} \mathbf{G} = u^*. \tag{7}$$

Since the kernel of Eq. (4) is equivalent to the gradient of a two-dimensional δ function as $h \to 0$, it may be shown that at a sufficient distance from the contour L with an accuracy up to small-order terms $O(h^2/d^2)$

$$\mathbf{G} = -h\Pi^{(2)} \operatorname{grad} u, \tag{8}$$

where

$$\Pi^{(2)} = \begin{Vmatrix} \frac{1}{2\pi}\int_0^{2\pi} \cos^2\psi f_2(\psi)\,d\psi & \frac{1}{2\pi}\int_0^{2\pi} \sin\psi\cos\psi f_2(\psi)\,d\psi \\ \frac{1}{2\pi}\int_0^{2\pi} \sin\psi\cos\psi f_2(\psi)\,d\psi & \frac{1}{2\pi}\int_0^{2\pi} \sin^2\psi f_2(\psi)\,d\psi \end{Vmatrix}; \tag{9}$$

$$f_2(\psi) = \frac{1}{2}\ln(1 + a^2/h^2).$$

Note that the difference in the corresponding components of the tensors $h^2\Pi^{(1)}$ and $h^2\Pi^{(2)}$ is of order $O(h^2/d^2)$.

With an error of the same order of smallness, the coefficient tensor may be replaced by an isotropic and homogeneous tensor

$$\Pi^{(3)} = \begin{pmatrix} \frac{1}{2}\ln d/h & 0 \\ 0 & \frac{1}{2}\ln d/h \end{pmatrix}. \tag{10}$$

Equation (8) resembles the Fick law $\mathbf{G} = -D \operatorname{grad} \rho$. The resemblance is purely external, however. In fact, the tensor $h\Pi^{(2)}$ differs in dimensionality from the tensor of diffusion coefficients D, and is a geometric characteristic of the region. The tensor $\Pi^{(\alpha)}$ ($\alpha = 1, 2, 3$) will be called the tensor of transmission coefficients, and the approximation corresponding to Eq. (8) will be said to be quasidiffusional. For long channels, this approximation was first considered in [2]. Below, the accuracy of the results obtained for various representation $\Pi^{(\alpha)}$ ($\alpha = 1, 2, 3$) of the transmission-coefficient tensor is compared.

To derive the zero solution of Eq. (5), the boundary condition on contour L must be specified. This condition may be obtained from integral Eq. (1) by Taylor-series expansion of the function u in the vicinity of point $\mathbf{y}_0$ lying on contour L

$$\frac{\partial u}{\partial n}(\mathbf{y}_0) = \frac{1}{h}[(-u(\mathbf{y}_0) + 2Q(\mathbf{y}_0))(1+\varkappa) + 2u^*], \tag{11}$$

where

$$\varkappa = \begin{cases} \dfrac{h}{2R(\mathbf{y}_0)}, & \text{if } \mathbf{y}_0 \text{ is a point of convexity of contour L,} \\ 0, & \text{if } \mathbf{y}_0 \text{ is a point of concavity of contour L.} \end{cases}$$

Thus, the problem of quasidiffusional approximation reduces to solving the Poisson equation

$$-h^2 \operatorname{div}(\Pi^{(\alpha)} \operatorname{grad} u) = u^* \tag{12}$$

in the region S with the boundary condition of Eq. (11) on contour L. Knowing u, and assuming that $v \simeq v^0$, it is simple to determine the effective fluxes at the walls and also the mass flux G inside region S from Eq. (8). The expression for G at points on contour L may be obtained from Eq. (4) by means of Taylor-series expansion of the unknown function u

$$G_n(\mathbf{y}_0) = \frac{1}{2}u(\mathbf{y}_0) - h\Pi_{ik}^{(\alpha)}(\mathbf{y}_0)\frac{\partial u}{\partial x_k}n_i - Q(\mathbf{y}_0), \tag{13}$$

where $\Pi_{ik}^{(\alpha)}$ are the components of the tensor $\Pi^{(\alpha)}$ (α = 1, 2, 3); n_i are the components of the vector **n**; summation from 1 to 2 is understood over the repeating subscripts i, k.

3. Error Estimate

Let $\hat{u}$ be the solution of integral Eq. (1) and u the solution of differential Eq. (5) with the boundary condition in Eq. (11). To simplify the estimates, the tensor $\Pi^{(\alpha)}$ will be considered in representation $\Pi^{(3)}$.

The function $\hat{u}$ satisfies Eq. (5) with the discrepancy $w(\mathbf{y})$

$$h^2\Pi\Delta\hat{u} = -u^* + w, \tag{14}$$

where $\Pi = (1/2)\ln(d/h)$; Δ is a two-dimensional Laplacian.

Taylor-series expansion of $\hat{u}$ in the integrand of Eq. (1) in the vicinity of the point **y** yields the form of the function $w(\mathbf{y})$

$$w(\mathbf{y}) = \frac{h^2}{2\pi}\int_0^{2\pi}\frac{(\hat{u}(\mathbf{y}) - 2Q(\psi))}{h^2 + a^2(\psi)}d\psi + h^2\frac{\partial\hat{u}}{\partial x}(\mathbf{y})\frac{1}{2\pi} \times \int_0^{2\pi}\cos\psi\left(-\frac{a(\psi)}{h^2 + a^2(\psi)} + \frac{1}{h}\arctan\frac{a(\psi)}{h}\right)d\psi + h^2\frac{\partial\hat{u}}{\partial y}(\mathbf{y})\frac{1}{2\pi} \times \int_0^{2\pi}\sin\psi\left(-\frac{a(\psi)}{h^2 + a^2(\psi)} + \frac{1}{h}\arctan\frac{a(\psi)}{h}\right)d\psi. \tag{15}$$

The absolute error $\Delta_u = \hat{u} - u$ satisfies the equation

$$h^2\Pi\Delta(\Delta_u) = w \tag{16}$$

and the boundary condition

$$h\frac{\partial\Delta_u}{\partial n} + \Delta_u(1+\varkappa) = 2\rho \quad \left(\rho(\mathbf{y}_0) \sim h^2\frac{\partial\hat{u}}{\partial n}\right). \tag{17}$$

solution Δ_u of Eqs. (16) and (17) is expressed in terms of its Green's function $g(\mathbf{y}, \mathbf{y}')$. Then

$$\Delta_u(\mathbf{y}) = \frac{2}{h}\int_L \rho(\mathbf{y}_0)g(\mathbf{y}_0, \mathbf{y})dL_{y_0} - \frac{1}{h^2\Pi}\int_S w(\mathbf{y}')g(\mathbf{y}', \mathbf{y})dS_{y'}. \tag{18}$$

The region S is divided into the boundary zone $\bar{S}$ and $S_0 = S \setminus \bar{S}$. Region $\bar{S}$ is of width

$l \ll \min_{y_0 \in L} R(y_0)$. The error will be estimated for points sufficiently far from the boundary of the region, i.e., for $y_0 \in S_0$. The second integral in Eq. (18) is divided into two integrals: with respect to S_0 and $\bar{S}$.

The integral $I_1 = \frac{1}{h^2\Pi} \int_{S_0} w(y') g(y', y) dS_{y'}$ is now estimated. It follows from Eq. (15) for $y' \in S_0$ that $w(y') \sim h^2 \max_L |\partial u/\partial n|$. It may also be shown that $\int_{S_0} g(y, y') dS_{y'}$ is finite as $h \to 0$. Thus

$$I_1 \sim \Pi^{-1} \max_L \left| \frac{\partial u}{\partial n} \right|. \tag{19}$$

Under the given assumptions, the integral

$$I_2 = \frac{1}{h^2\Pi} \int_{\bar{S}} w(y') g(y', y) dS_{y'} \approx \frac{L}{h^2\Pi} \int_0^l w(x) g(x, y) dx.$$

The boundary condition for the Green's function $g(y', y)$ takes the form

$$h \frac{\partial g}{\partial x}(0, y) + g(0, y)(1 + \varkappa) = 0. \tag{20}$$

As follows from Eq. (20), $I_2 \sim \frac{L}{h^2\Pi} \int_0^l w(x)(x+h) dx$. To estimate $w(x)$ close to the boundary, Eq. (15) is used, replacing the integral over the contour L by an integral over the adjacent vicinity. It may be shown that the error resulting from this substitution is no more than $O((h/d)^2 \ln (h/d))$. The corresponding calculations give the result

$$I_2 \sim \Pi^{-1} \max_L \left| \frac{\partial u}{\partial n} \right|. \tag{21}$$

It may now be noted that the first term in Eq. (18) is of order $h \max_L |\partial u/\partial n|$. It follows from Eqs. (18), (19), and (21) that $\Delta_u(y) \sim \Pi^{-1} \max_L |\partial u/\partial n|$ for $y \in S_0$. The relative error δ_u in determining the function u is

$$\delta_u = \Delta_u(y) \cdot \left(\max_L \left| \frac{\partial u}{\partial n} \right| d \right)^{-1} \sim \Pi^{-1}.$$

The error in determining the function v is now estimated. Let $\hat{v}$ be the solution of integral Eq. (2). Then the absolute error $\Delta_v = \hat{v} - (v^*/2)$ satisfies the integral equation

$$\Delta_v + \int_S \Delta_v(y') K(y, y') dS_{y'} = \frac{v^*}{2} \left[1 - \int_S K(y, y') dS_{y'} \right]. \tag{22}$$

Taylor-series expansion of $\Delta_v(y')$ in the integrand in the vicinity of point y shows that far from the boundary

$$\Delta_v(y) = \left(\Delta_v(y) + \frac{v^*}{2} \right) \frac{h^2}{4\pi} \int_0^{2\pi} \frac{d\psi}{h^2 + a^2(\psi)} - \frac{\partial \Delta_v}{\partial x}(y) \frac{h^2}{4\pi}$$

$$\times \int_0^{2\pi} \cos\psi \left(\frac{a(\psi)}{h^2 + a^2(\psi)} + \frac{1}{a(\psi)} \right) d\psi - \frac{\partial \Delta_v}{\partial y}(y) \frac{h^2}{4\pi} \int_0^{2\pi} \sin\psi \left(\frac{a(\psi)}{h^2 + a^2(\psi)} + \frac{1}{a(\psi)} \right) d\psi. \tag{23}$$

It may be concluded from Eq. (23) that the relative error in determining the function v is $\delta_v = \Delta_v (d \max_{S_0} |\mathrm{grad}\, v|)^{-1} = O(h^2/d^2)$.

It follows from Eq. (3) that the relative error in determining the effective fluxes q_1 and q_2 is of order Π^{-1}. The error in determining the mass fluxes in the gap is estimated analogously. It is also found here that the relative error is of the order of Π^{-1}.

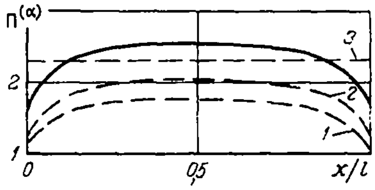

Fig. 1.

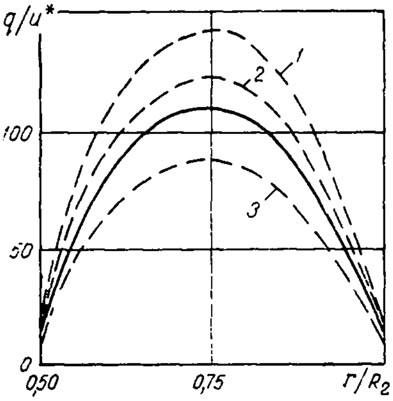

Fig. 2.

Fig. 1. Dependence of the values of the transmission coefficient on the dimensionless ratio x/l for an infinite strip.

Fig. 2. Dependence of the dimensionless effective flux on the ratio r/R_2 for an annular gap with $u^* \neq 0$.

4. Particular Case of a Gap between Two Annular Disks

As an example, consider the gap between two annular disks. The region S is described by the conditions $R_1 < r < R_2$, $0 \leqslant \varphi \leqslant 2\pi$. in the polar coordinates (r, φ). The quantities q_j depend solely on r, and the fluxes $Q_1 = Q|_{r=R_1}$ and $Q_2 = Q|_{r=R_2}$ are constant. Assuming that $R_2 = R_1 + l$, $r = R_1 + x$, and letting R_1 tend to infinity, an infinite strip bounded by the two straight lines $x = 0$ and $x = l$ is obtained as the region S, in the limit. It is obvious that $Q_1 = Q|_{x=0}$ and $Q_2 = Q|_{x=l}$. The effective flux densities q_j and the function u are then constant over the whole length of the strip and depend only on the coordinate x.

Since the coordinate axes in the given problem are the principal axes of the transmission tensor $\Pi^{(\alpha)}$, only the components $\Pi_{11}^{(\alpha)}$ and $\Pi_{22}^{(\alpha)}$ are nonzero. In view of the one-dimensionality of the problem, the only quantities which play a role here are $\Pi^{(\alpha)} = \Pi_{11}^{(\alpha)}$, which are functions of a single coordinate.

For an annular gap, the transmission coefficients take the form

$$\Pi^{(1)}(r) = \frac{1}{2\pi}\int_{R_1}^{R_2}(r-r_1)^2 K_1(r, r_1)\,dr_1, \quad \Pi^{(2)}(r) = \frac{1}{\pi h^2}\int_{R_1}^{R_2}(r-r_1)V(r, r_1)\,dr_1,$$

$$\Pi^{(3)} = \frac{1}{2}\ln\frac{R_2-R_1}{h}, \tag{24}$$

where

$$K_1(r, r_1) = \frac{4r_1(h^2+r^2+r_1^2)}{(AB)^{3/2}}\arctan\left(\sqrt{\frac{A}{B}}Z\right) + \frac{8r_1^2 rZ}{AB(AZ^2+B)};$$

$$V(r, r_1) = \frac{r_1}{r}\left[\frac{-r^2+r_1^2+h^2}{(AB)^{1/2}}\arctan\left(\sqrt{\frac{A}{B}}Z\right) + \arctan\left(\frac{r+r_1}{r-r_1}Z\right)\right];$$

$$A = h^2+(r+r_1)^2;\ B = h^2+(r-r_1)^2;\ Z = \frac{r\sqrt{r_1^2-R_1^2}+r_1\sqrt{r^2-R_1^2}}{R_1(r+r_1)}.$$

The general form of the solution for an annular gap is

$$u(r) = -\frac{u^*}{2h^2}\int_{R_1}^{r}\frac{r'dr'}{\Pi^{(\alpha)}(r')} + \frac{c}{h}\int_{R_1}^{r}\frac{dr'}{r'\Pi^{(\alpha)}(r')} + b. \tag{25}$$

The boundary condition in Eq. (11) for Eq. (25) takes the form

$$\frac{\partial u}{\partial r}(R_1) = \frac{1}{h}[u(R_1) - 2Q_1 - 2u^*],$$

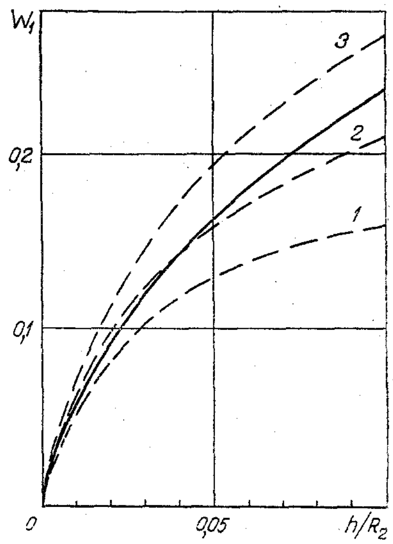

Fig. 3.

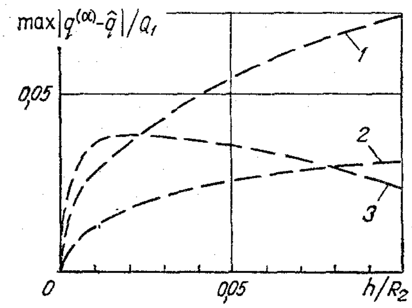

Fig. 4.

Fig. 3. Dependence of the probability W_1 on the dimensionless magnitude of the gap.

Fig. 4. Dependence of the relative error in determining the effective flux on the dimensionless magnitude of the gap.

$$-\frac{\partial u}{\partial r}(R_2)=\frac{1}{h}\left[(u(R_2)-2Q_2)\left(1+\frac{h}{2R_2}\right)-2u^*\right]. \tag{26}$$

In the limiting case of an infinite strip, the transmission coefficients are taken in the form

$$\Pi^{(1)}(x)=\frac{1}{4}\left[\ln\frac{\sqrt{h^2+(l-x)^2}+l-x}{\sqrt{h^2+x^2}-x}-\frac{l-x}{\sqrt{h^2+(l-x)^2}}-\frac{x}{\sqrt{h^2+x^2}}\right],$$

$$\Pi^{(2)}(x)=\frac{1}{4}\left[\ln\frac{\sqrt{h^2+(l-x)^2}+l-x}{\sqrt{h^2+x^2}-x}-\frac{l-x}{\sqrt{h^2+(l-x)^2}+l-x}-\frac{x}{\sqrt{h^2+x^2}+x}\right], \tag{27}$$

$$\Pi^{(3)}=\frac{1}{2}\ln(l/h).$$

The general solution of Eq. (12) takes the following form in the case of an infinite strip

$$u(x)=-\frac{u^*}{h^2}\int\limits_0^x\frac{x'dx'}{\Pi^{(\alpha)}(x')}+\frac{c}{h^2}\int\limits_0^x\frac{dx'}{\Pi^{(\alpha)}(x')}+b. \tag{28}$$

The boundary condition in Eq. (11) then takes the form

$$-\frac{\partial u}{\partial x}(0)=\frac{1}{h}[-u(0)+2Q_1+2u^*],\quad \frac{\partial u}{\partial x}(l)=\frac{1}{h}[-u(l)+2Q_2+2u^*], \tag{29}$$

since the radius of curvature of contour L is infinite in this case. The boundary conditions in Eqs. (26) and (29) offer the possibility of determining the constants c and b in Eqs. (25) and (28).

Knowing u and G_n, quantities characterizing the conduction of the annular gap may be found: the probabilities that a flux incident on the external and internal cylindrical surface, respectively, will pass through the channel

$$W_1=-\frac{R_1G_n(R_1)}{R_2Q_2}\quad\text{for}\quad u^*=0,\ Q_1=0, \tag{30}$$

$$W_2=\frac{R_2G_n(R_2)}{R_1Q_1}\quad\text{for}\quad u^*=0,\ Q_2=0, \tag{31}$$

and also the probabilities that the emitted flux will exit through the internal and external

TABLE 1. Values of the Ratio $q(r)/Q_1$ when $q^* = 0$, $Q_2 = 0$, $R_1/R_2 = 0.5$, $h/R_2 = 0.01$

Ratio r/R_2	0,5	0,625	0,75	0,875	1,0
Numerical solution of integral equation	0,957	0,628	0,397	0,205	0,020
Analytical solution of boundary problem for $\Pi^{(\alpha)} = \Pi^{(1)}$	0,964	0,648	0,425	0,230	0,021
$\Pi^{(\alpha)} = \Pi^{(2)}$	0,950	0,634	0,409	0,217	0,021
Same, for $\Pi^{(\alpha)} = \Pi^{(3)}$	0,972	0,664	0,412	0,198	0,014

cylindrical surface, respectively,

$$W_1^* = -\frac{2R_1 h G_n(R_1)}{(R_2^2 - R_1^2)u^*} \quad \text{for} \quad Q_1 = Q_2 = 0, \tag{32}$$

$$W_2^* = \frac{2R_2 h G_n(R_2)}{(R_2^2 - R_1^2)u^*} \quad \text{for} \quad Q_1 = Q_2 = 0. \tag{33}$$

Note that $W_1^* + W_2^* = 1$. In the limiting case of an infinite strip, $W_1 = W_2$ and $W_1^* = W_2^* = 0.5$ in view of the symmetry of the region. Hence, for an infinite strip, it is reasonable to introduce a single quantity W, the probability that the flux incident on the surface x = 0 from outside will pass through the gap:

$$W = \frac{G_n(l)}{Q_1} \quad \text{for} \quad u^* = 0,\ Q_2 = 0. \tag{34}$$

5. Results of Numerical Comparison

In the case of a gap between two annular disks, and in the limiting case of an infinite strip, a numerical solution of integral Eq. (1) has been found by the Krylov–Bogolyubov method [4]. The accuracy with which it approximates the analytical solution of the problem of the quasidiffusional approximation is compared for three different transmission coefficients $\Pi^{(\alpha)}$. The continuous curves in Figs. 1-4 show the numerical solution and the dashed curves the analytical solution. The numbers on the curve correspond to $\alpha = 1, 2, 3$.

Numerical solution offers the possibility, in particular, of determining the dependence of the transmission coefficient given by Eq. (8) on the coordinate. This dependence is shown in Fig. 1 in comparison with the functions $\Pi^{(\alpha)}$. As is evident from Fig. 1, the best approximation to the numerical value of the transmission coefficient is given by $\Pi^{(3)}$ on average and by $\Pi^{(2)}$ at the edges of the region. Values of the effective flux are given in Fig. 2 and Table 1. The coefficient $\Pi^{(2)}$ gives the best approximation in both the case $u^* = 0$ and the case $u^* \neq 0$.

The probabilities that the flow will pass through the annular gap defined in Eq. (30) are shown in Fig. 3 as a function of h. It is evident that the value of W_1 is most accurately approximated by the analytical result when $\Pi = \Pi^{(2)}$, if h is not very small. However, when $h/d \leqslant 0.02$, transmission coefficient $\Pi^{(1)}$ gives the best approximation. For the probability W_1^* defined in Eq. (32), $\Pi^{(2)}$ and $\Pi^{(3)}$ give equally good results (no more than 2-3% error).

Values of the error in calculating the effective flux in an annular gap for different forms of the transmission tensor are shown in Fig. 4. In this case, the smallest error when $h/d \leq 0.07$ corresponds to the approximation with transmission coefficient $\Pi^{(2)}$ and when $h/d \geqslant 0.07$ to $\Pi^{(3)}$. The relative error is no more than 3% here. Note that, although the overall estimate of the error – ($\delta_q \sim (\ln(d/h))^{-1}$) – is not very optimistic, an error that is perfectly acceptable for practical purposes is obtained even at relatively small values of ln d/h.

NOTATION

$a(\psi)$, distance from point **y** to contour L in the direction forming an angle $\pi + \psi$ with the Ox axis; b, c, arbitrary constants; D, diffusion coefficient; d, characteristic dimension of region; **G**, two-dimensional mass-flux-density vector along the gap; $g(\mathbf{y}, \mathbf{y}')$, Green's function; h, distance between the walls of the gap; L, contour of the region S; l, width of infinite strip; **n**, unit vector normal to contour L; Q, Q_1, Q_2, flux densities incident from outside; q_1, q_2, effective flux densities at walls; q_1^*, q_2^*, densities of emitted fluxes at walls; $R(y_0)$, radius of curvature of contour L at point y_0; R_1, R_2, internal and external radii of annular gap; r, coordinate; S, two-dimensional region; $u = q_1 + q_2$, $v = q_2 - q_1$; W_1, W_2, probabilities that a flux incident respectively on the external and internal cylindrical surface will pass through the gap; W_1^*, W_2^*, probabilities that the emitted flux will exit through the internal and external cylindrical surface, respectively; w, discrepancy in the equation; x, coordinate; y, point of region S; Δ, Laplacian; Δ_u, Δ_v, absolute errors; δ_u, δ_v, relative errors; $\Pi^{(\alpha)}$, transmission-coefficient tensor. Indices: α, number of the representation for the transmission coefficient; j, number of gap wall.

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NEW APPROXIMATE ANALYTIC METHODS OF INVESTIGATING PROBLEMS OF PHYSICOCHEMICAL MECHANICS

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New approximate analytic methods are suggested for investigating problems of physicochemical mechanics. Specific examples are provided, illustrating the use of these methods.

1. Asymptotic Correction Method. Various engineering equations, obtained empirically or by approximate solution of the corresponding (boundary-value) problems, are often used in practice. The validity region of these equations is usually restricted, and is separately established in each specific case. Below we suggest a simple universal method of substantial improvement of the approximate engineering equations, based on using the exact asymptotic of the original boundary-value problem.

Let the unknown quantity S be obtained by the approximate expression

$$S = S(k,\ P), \tag{1}$$

which usually reflects the qualitative behavior of S as a function of the change in the dominant parameters of the problem k and P (here and later it is assumed for simplicity that there are two such parameters). Let the main terms of the asymptotic approximate expression (1) be in the limiting cases $k \to \infty$ (P = const) and $P \to \infty$ (k = const)

$$k \to \infty,\ S \to S^*_{\infty P};\ P \to \infty,\ S \to S^*_{k\infty}; \tag{2}$$

$$S^*_{\infty P} = S^*_{\infty P}(k,\ P),\ S^*_{k\infty} = S^*_{k\infty}(k,\ P) \tag{3}$$

(instead of (2) one can consider any other limiting cases; see the specific examples provided below).

If similar exact asymptotic solutions of the original problem are known

$$k \to \infty,\ S \to S_{\infty P};\ P \to \infty,\ S \to S_{k\infty}, \tag{4}$$

the approximate Eq. (1) can be improved by the following simple method. In expression (3) we

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